Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Functions:

Transformations and Graphs

Calculators may NOT be used for these questions.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' might be needed for some questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are 15 questions in this test.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear.

Answers without working may not gain full credit.

1.
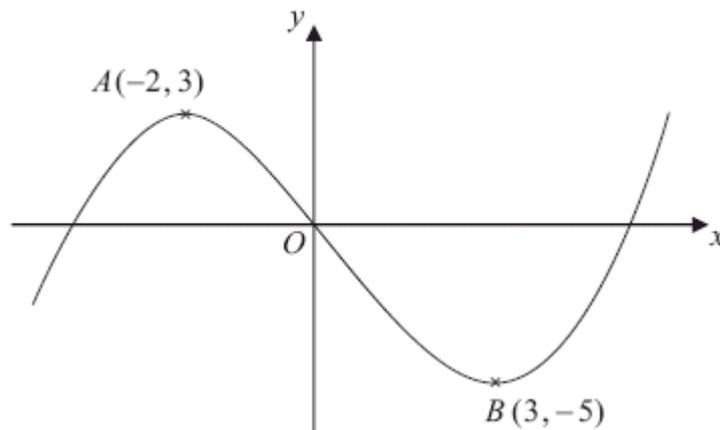
The diagram above shows a sketch of the curve with equation $y = f(x)$. The curve has a maximum point A at $(-2, 3)$ and a minimum point B at $(3, -5)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 3)$

(3)

(b) $y = 2f(x)$

(3)

On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = f(x) + a$ has a minimum at $(3, 0)$, where a is a constant.

(c) Write down the value of a.

(1)
(Total 7 marks)

_____

2.   (a)   On the axes below sketch the graphs of

(i) $y = x(4 - x)$

(ii) $y = x^2(7 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.
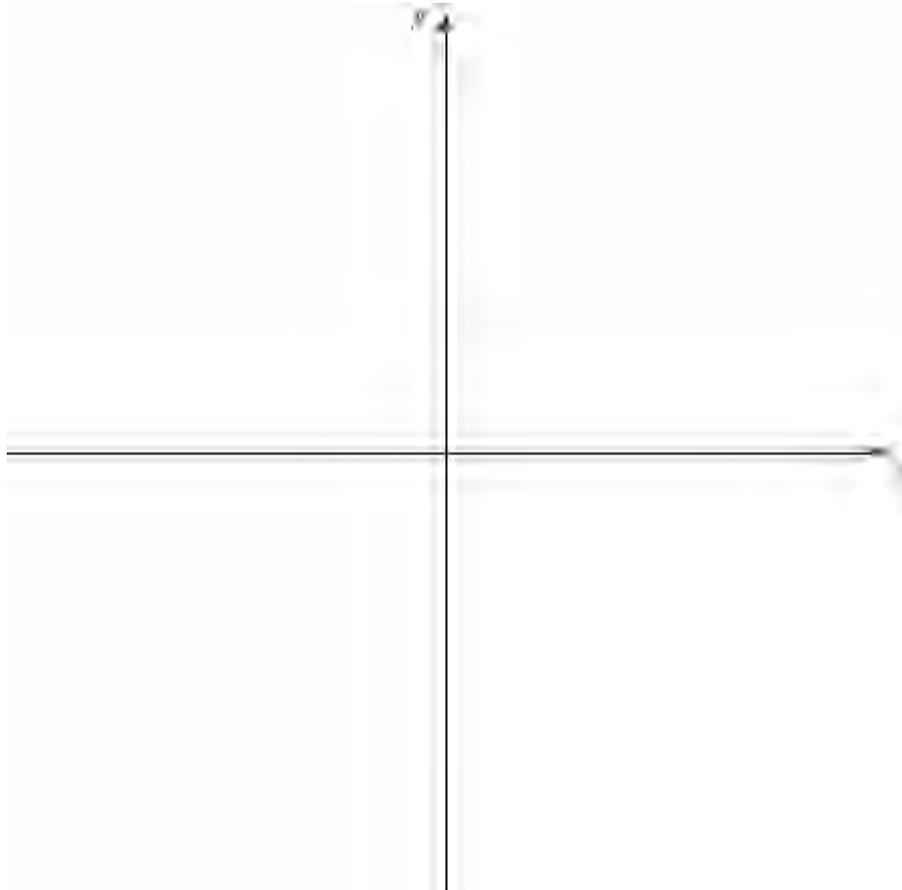
(5)

(b)   Show that the *x*-coordinates of the points of intersection of

$$y = x(4 - x) \text{ and } y = x^2(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point *A* lies on both of the curves and the *x* and *y* coordinates of *A* are both positive.

(c)   Find the exact coordinates of *A*, leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where *p*, *q*, *r* and *s* are integers.

(7)
(Total 15 marks)

_____

3.
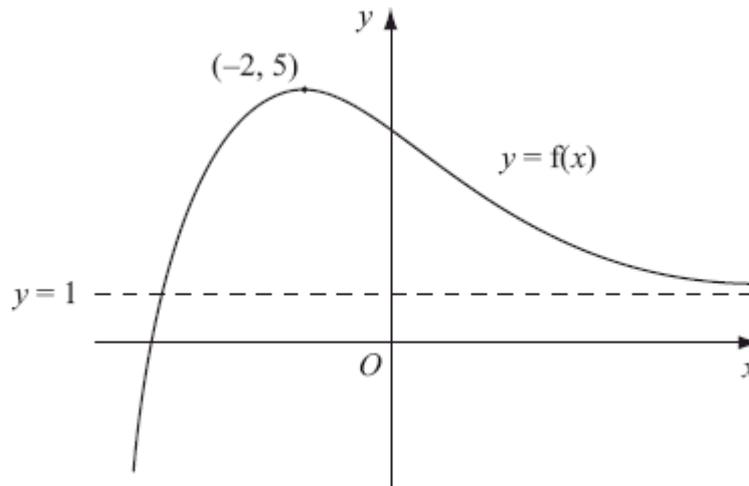
The diagram above shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

The curve has a maximum point $(-2, 5)$ and an asymptote $y = 1$, as shown in the diagram.

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x) + 2$

(2)

(b) $y = 4\mathrm{f}(x)$

(2)

(c) $y = \mathrm{f}(x + 1)$

(3)

On each diagram, show clearly the coordinates of the maximum point and the equation of the asymptote.

(Total 7 marks)

4.   (a)   Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b)   Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c)   sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the *x*-axis.

(2)
(Total 9 marks)

_____

5.

(2, −1)
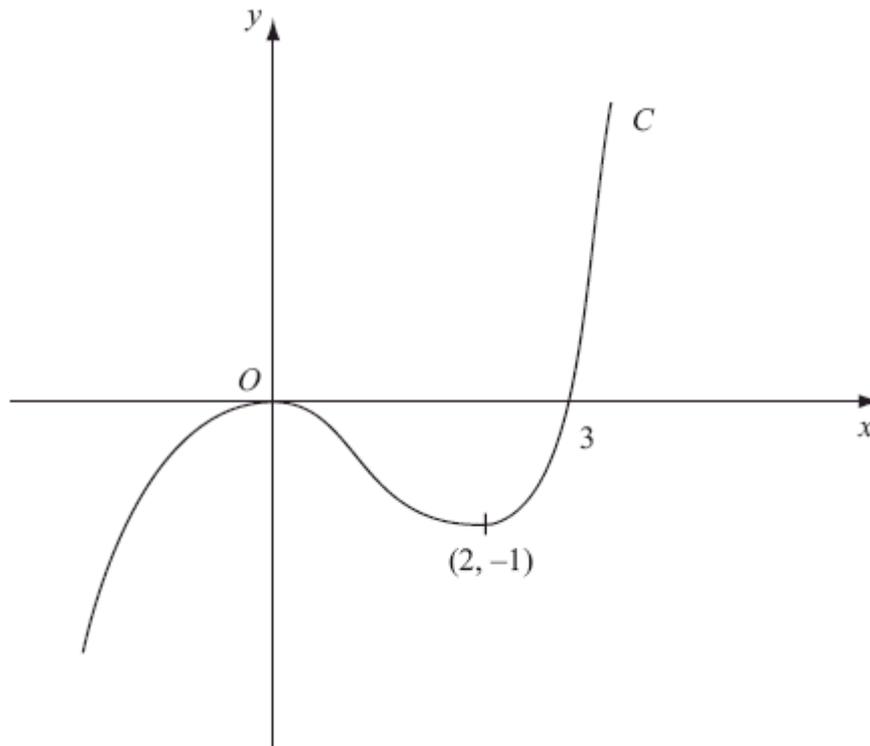
Above is a sketch of the curve C with equation $y = $ f(x). There is a maximum at $(0, 0)$, a minimum at $(2, −1)$ and C passes through $(3, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = $ f$(x + 3)$

(3)

(b) $y = $ f$(−x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

(Total 6 marks)

_____

6.
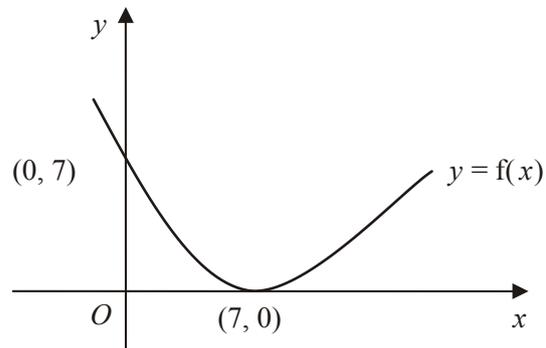
The diagram above shows a sketch of the curve with equation $y = f(x)$. The curve passes through the point (0, 7) and has a minimum point at (7, 0).

On separate diagrams, sketch the curve with equation

(a) $y = f(x) + 3$,

(3)

(b) $y = f(2x)$.

(2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the *y*-axis.

(Total 5 marks)

_____

7.
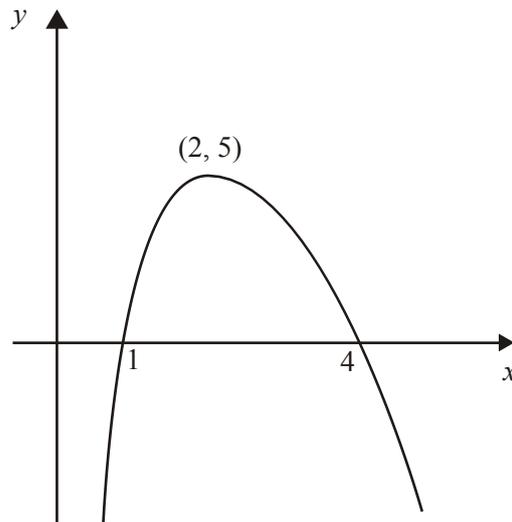
The diagram above shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (1, 0) and (4, 0). The maximum point on the curve is (2, 5). In separate diagrams sketch the curves with the following equations. On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2f(x)$,

(3)

(b) $y = f(-x)$.

(3)

The maximum point on the curve with equation $y = f(x + a)$ is on the y-axis.

(c)   Write down the value of the constant a.

(1)
(Total 7 marks)

_____

8.
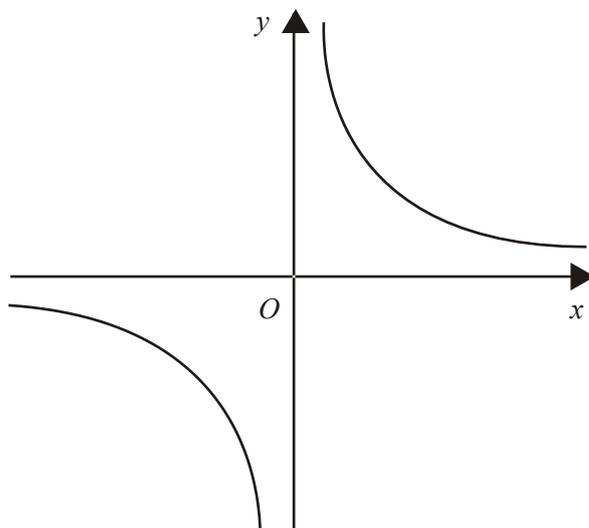
The diagram above shows a sketch of the curve with equation $y = \dfrac{3}{x}, \ x \neq 0.$

(a)   On a separate diagram, sketch the curve with equation $y = \dfrac{3}{x+2}, \ x \neq -2,$ showing

the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b)   Write down the equations of the asymptotes of the curve in part (a).

(2)
(Total 5 marks)

_____

9.   Given that $f(x) = \dfrac{1}{x}, \quad x \neq 0,$

(a)   sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes.

(4)

(b)   Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis.

(2)
(Total 6 marks)

_____

10.   (a)   On the same axes sketch the graphs of the curves with equations

(i) $y = x^2(x - 2)$,

(3)

(ii) $y = x(6 - x)$,

(3)

and indicate on your sketches the coordinates of all the points where the curves cross the *x*-axis.

(b)   Use algebra to find the coordinates of the points where the graphs intersect.

(7)
(Total 13 marks)

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11.   On separate diagrams, sketch the graphs of

(a) $y = (x + 3)^2$,

(3)

(b) $y = (x + 3)^2 + k$, where *k* is a positive constant.

(2)

Show on each sketch the coordinates of each point at which the graph meets the axes.
(Total 5 marks)

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12.
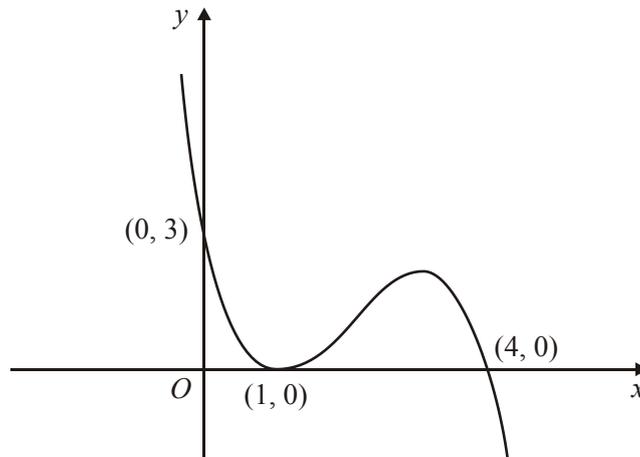
The figure above shows a sketch of the curve with equation $y = f(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams sketch the curve with equation

(a) $y = f(x + 1)$,

(3)

(b) $y = 2f(x)$,

(3)

(c) $y = f\left(\dfrac{1}{2}x\right)$.

(3)

On each diagram show clearly the coordinates of all the points where the curve meets the axes.

(Total 9 marks)

_____

13.
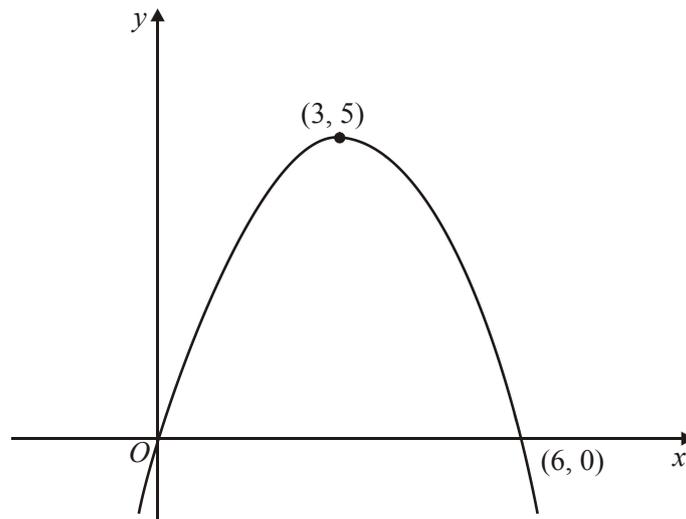
The diagram shows a sketch of the curve with equation $y = f(x)$. The curve passes through the origin O and through the point $(6, 0)$. The maximum point on the curve is $(3, 5)$.

On separate diagrams, sketch the curve with equation

(a) $y = 3f(x)$,

(2)

(b) $y = f(x + 2)$.

(3)

On each diagram, show clearly the coordinates of the maximum point and of each point at which the curve crosses the *x*-axis.

(Total 5 marks)

_____

14.
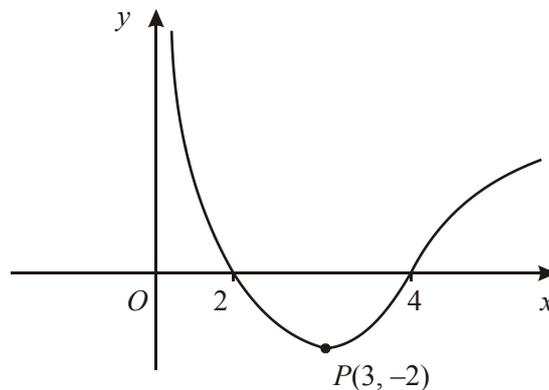
This diagram shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points $(2, 0)$ and $(4, 0)$. The minimum point on the curve is $P(3, -2)$.

In separate diagrams sketch the curve with equation

(a) $y = -f(x)$,

(3)

(b) $y = f(2x)$.

(3)

On each diagram, give the coordinates of the points at which the curve crosses the x-axis, and the coordinates of the image of P under the given transformation.

(Total 6 marks)

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15.   (a)   Factorise completely $x^3 - 4x$.

(3)

(b)   Sketch the curve with equation $y = x^3 - 4x$, showing the coordinates of the points where the curve crosses the x-axis.

(3)

 (c)   On a separate diagram, sketch the curve with equation

$$y = (x - 1)^3 - 4(x - 1),$$

showing the coordinates of the points where the curve crosses the x-axis.

(3)
(Total 9 marks)

___

1. (a)
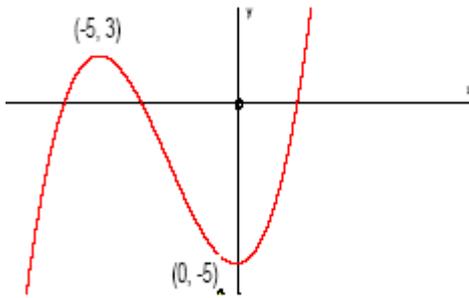
Horizontal translation of ± 3	M1
$(-5, 3)$ marked on sketch **or in text**	B1
$(0, -5)$ and min intentionally on *y*-axis	A1 3

Condone $(-5, 0)$ if correctly placed on negative *y*-axis

Note

Turning points (not on axes) should have both co-ordinates given in form (x, y). Do not accept points marked on axes e.g. -5 on *x*-axis and 3 on *y*-axis is not sufficient. For repeated offenders apply this penalty **once only** at first offence and condone elsewhere.

In (a) and (b) no graphs means no marks.

In (a) and (b) the ends of the graphs do not need to cross the axes provided max and min are clear
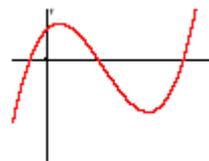
M1 for a horizontal translation of ± 3 so accept
i.e max in 1$^{\text{st}}$ quad <u>and</u>
coordinates of $(1, 3)$ <u>or</u> $(6, -5)$ seen.
[Horizontal translation to the left should have a min <u>on</u> the *y*-axis]
If curve passes through $(0, 0)$ then M0 (and A0) but they
could score the B1 mark.

A1 for minimum clearly on negative *y*-axis and at least -5 marked on
y-axis.
Allow this mark if the minimum is very close and the point
$(0, -5)$ clearly indicated

(b)
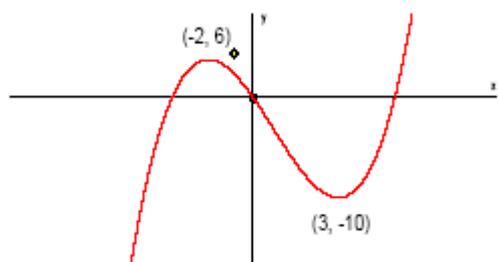
Correct shape and intentionally through (0, 0) between the max and min   B1

$(-2, 6)$ marked on graph **or in text**   B1

$(3, -10)$ marked on graph **or in text**   B1   3

Note

Turning points (not on axes) should have both co-ordinates given in form(x, y). Do not accept points marked on axes e.g. -5 on x-axis and 3 on y-axis is not sufficient. For repeated offenders apply this penalty **once only** at first offence and condone elsewhere.

In (a) and (b) no graphs means no marks.

In (a) and (b) the ends of the graphs do not need to cross the axes provided max and min are clear

1st B1 Ignore coordinates for this mark
    Coordinates or points on sketch override coordinates given in the text.
    Condone (y, x) confusion for points on axes only. So $(-5, 0)$ for
    $(0, -5)$ is OK if the point is marked correctly but $(3, 10)$ is B0
    even if in 4th quadrant.

(c) $(a =)\ \underline{5}$ B1   1

Note

**This may be at the bottom of a page or in the question...make sure
you scroll up and down!**

[7]

2.   (a)
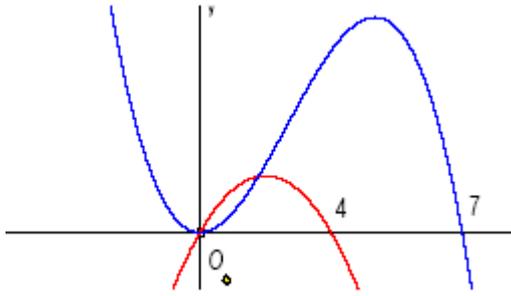
(i)   ∩ shape (anywhere on diagram)   B1

Passing through or stopping at (0, 0) and (4, 0)   B1
only(Needn't be ∩ shape)

(ii)   correct shape (-ve cubic) with a max and min drawn anywhere B1
Minimum or maximum at (0, 0)   B1
Passes through or stops at (7, 0) but <u>NOT</u> touching.   B1   5
(7, 0) should be to right of (4, 0) or B0
Condone (0, 4) or (0, 7) marked correctly on *x*-axis.
Don't penalise poor overlap near origin.
Points must be marked on the sketch...not in the text

(b) $x(4-x) = x^2(7-x)$ $(0=)x[7x - x^2 - (4-x)]$ M1

$(0=)x[7x - x^2 - (4-x)]$ (o.e.)   B1ft

$0 = x(x^2 - 8x + 4)$ *   A1 cso   3

<u>Note</u>

M1   for forming a suitable equation
B1   for a common factor of *x* taken out legitimately. Can treat this
as an M mark. Can ft their cubic = 0 found from an attempt at
solving their equations e.g. $x^3 - 8x^2 - 4x = x(...$
A1cso   no incorrect working seen. The "= 0" is required but condone
missing from some lines of working. Cancelling the *x* scores B0A0.

(c) $\left(0 = x^2 - 8x + 4 \Rightarrow\right)x = \dfrac{8 \pm \sqrt{64-16}}{2}$ or $\begin{matrix}(x \pm 4)^2 - 4^2 + 4(=0) \\ (x-4)^2 = 12\end{matrix}$ M1

A1

$= \dfrac{8 \pm 4\sqrt{3}}{2}$ or $(x-4) = \pm 2\sqrt{3}$ B1

$x = 4 \pm 2\sqrt{3}$ A1

From sketch *A* is $x = 4 - 2\sqrt{3}$ M1

So $y = \left(4 - 2\sqrt{3}\right)\left(4 - [4 - 2\sqrt{3}]\right)$ (dependent on 1st M1) M1

$\qquad\qquad\qquad\quad = -12 + 8\sqrt{3}$ A1   7

Note

1st M1   for some use of the correct formula or attempt to complete the square

1st A1   for a fully correct expression: condone + instead of ±
     or for $(x - 4)^2 = 12$

B1   for simplifying $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{12} = 2\sqrt{3}$ .Can be scored independently of this expression

2nd A1   for correct solution of the form $p + q\sqrt{3}$: can be ± or + or −

2nd M1   for selecting their answer in the interval (0, 4). If they have no value in (0, 4) score M0

3rd M1   for attempting $y = \ldots$ using their x in correct equation. An expression needed for M1A0

3rd A1   for correct answer. If 2 answers are given A0.

[15]

3.   (a)
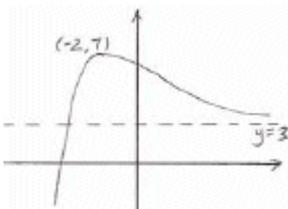
(−2, 7), $y = 3$ (Marks are dependent upon a sketch
                            being attempted)   B1 B1   2

            See conditions below.

      (b)
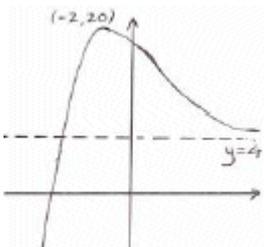
(−2, 20), $y = 4$ (Marks are dependent upon a sketch
                            being attempted)   B1 B1   2

            See conditions below.

Note

Parts (a) and (b):

(i) If <u>only one</u> of the B marks is scored, there is <u>no penalty</u> for a wrong sketch.

(ii) If both the maximum and the equation of the asymptote are correct, the sketch must be "correct" to score B1 B1.
If the sketch is "wrong", award B1 B0.
The (generous) conditions for a "correct" sketch are that the maximum must be in the 2nd quadrant and that the curve must not cross the positive x-axis... ignore other "errors" such as "curve appearing to cross its asymptote" and "curve appearing to have a minimum in the 1st quadrant".

Special case:

(b)   Stretch $\frac{1}{4}$ instead of 4: Correct shape, with $\left(-2, \frac{5}{4}\right)$, $y = \frac{1}{4}$: B1 B0

Coordinates of maximum:

If the coordinates are the wrong way round (e.g. $(7, -2)$ in part (a)), or the coordinates are just shown as values on the x and y axes, penalise <u>only once in the whole question</u>, at first occurrence.

Asymptote marks:

If the <u>equation</u> of the asymptote is not given,

e.g. in part (a), 3 is marked on the y-axis but $y = 3$ is not seen, penalise <u>only once in the whole question</u>, at first occurrence.

<u>Ignore</u> extra asymptotes stated (such as $x = 0$).

(c)
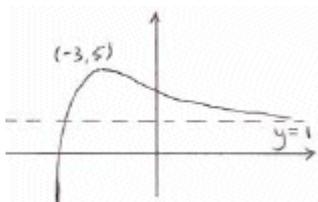
Sketch: Horizontal translation (either way)… (There must be evidence that $y = 5$ at the max and that the asymptote is still $y = 1$) B1

$(-3, 5)$, $y = 1$ B1 B1   3

[7]

4.   (a) $x(x^2 – 6x + 9)$ B1
            $= x(x – 3)(x – 3)$ M1 A1   3

Note

B1   for correctly taking out a factor of x

M1   for an attempt to factorize their 3TQ e.g. $(x + p)(x + q)$
        where $|pq| = 9$.
        So $(x – 3)(x + 3)$ will score M1 but A0

A1   for a fully correct factorized expression – accept $x(x – 3)^2$
        If they "solve" use ISW

S.C.

        If the only correct linear factor is $(x – 3)$, perhaps from factor
theorem, award B0M1A0
        Do not award marks for factorising in part (b)

For the graphs

        "Sharp points" will lose the 1st B1 in (b) but otherwise be
        generous on shape Condone $(0, 3)$ in (b) and $(0, 2)$, $(0,5)$ in (c) if
        the points are marked in the correct places.

(b)
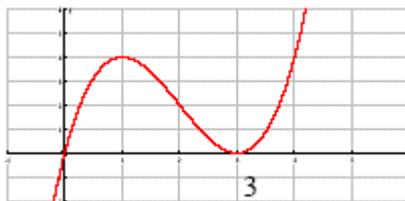
Shape $\sim\!\!\!J$
Through origin (not touching)   B1
Touching x-axis only once   B1
Touching at $(3, 0)$, or 3 on x-axis   B1ft   4
[Must be on graph not in a table]

Note

2nd B1 for a curve that starts or terminates at $(0, 0)$ score B0

4th B1ft for a curve that touches (not crossing or terminating) at $(a, 0)$
where their $y = x(x – a)^2$

(c)
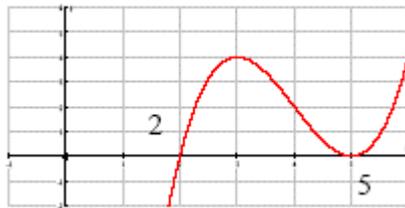
Moved horizontally (either way)	M1	
(2, 0) and (5, 0), or 2 and 5 on x-axis	A1	2

<u>Note</u>

M1   for their graph moved horizontally (only) <u>or</u> a fully correct graph
     Condone a partial stretch if ignoring their values looks like a simple translation

A1   for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only

     Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)

[9]

5.   (a)
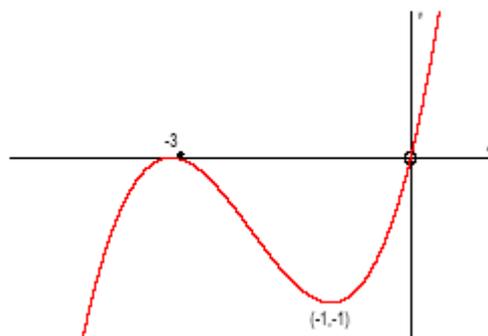
Shape $\bigwedge\!\!\bigvee$, touching the *x*-axis at its maximum.	M1	
Through (0, 0) & − 3 marked on x-axis, or (−3,0) seen.	A1	
Allow (0, − 3) if marked on the *x*-axis.		
Marked in the correct place, but 3, is A0.		
Min at (−1, −1)	A1	3

<u>Note</u>

M1 as described above. Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min.

1st A1   for curve passing through − 3 and the origin. Max at (− 3,0)

2nd A1 for minimum at (− 1, − 1). Can simply be indicated on sketch.

(b)
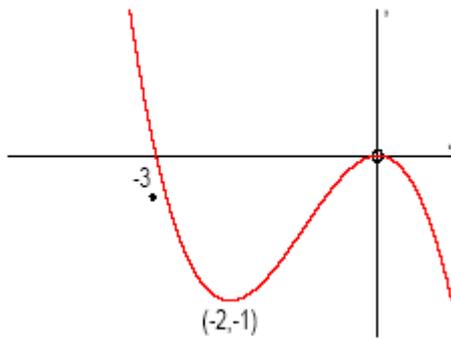
Correct shape ⋀⋁
(top left – bottom right)   B1

Through – 3 and max at (0, 0).
Marked in the correct place, but 3, is B0.   B1

Min at (–2, –1)   B1   3

Note

1st B1 for the correct shape. A negative cubic passing from top left to
       bottom right. Shape: Be generous, even when the curve seems
       to be composed of straight line segments, but there must be a
       discernible 'curve' at the max. and min.

2nd B1 for curve passing through (–3,0) having a max at (0, 0) and no
        other max.

3rd B1 for minimum at (–2, –1) and no other minimum. If in correct
       quadrant but labelled, e.g. (–2,1), this is B0.

In each part the (0, 0) does <u>not</u> need to be written to score the second
mark… having the curve pass through the origin is sufficient.

The last mark (for the minimum) in each part is dependent on a sketch
being attempted, and the sketch must show the minimum in approximately
the correct place (not, for example, (–2, –1) marked in the wrong quadrant).
The mark for the minimum is <u>not</u> given for the coordinates just marked on
the axes <u>unless</u> these are clearly linked to the minimum by vertical and
horizontal lines.

[6]

6.   (a)
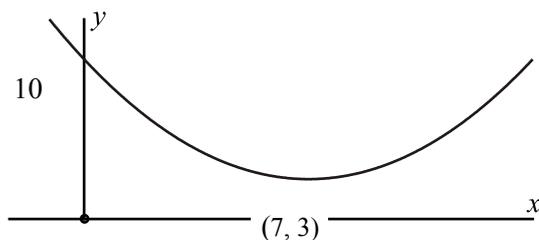
$(7, 3)$ B1B1B1   3

1st B1   for moving the given curve up. Must be U shaped curve, minimum in first quadrant, not touching *x*-axis but cutting positive *y*-axis. Ignore any values on axes.

2nd B1   for curve cutting *y*-axis at (0, 10). Point 10(or even (10, 0) marked on positive *y*-axis is OK)

3rd B1   for minimum indicated at (7, 3). Must have both coordinates and in the right order.
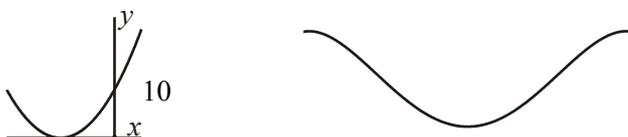
this would score B0B1B0

If the curve flattens out to a turning point like this penalise <u>once</u> at first offence ie 1st B1 in (a) or in (b) but not in both.

The U shape mark can be awarded if the sides are fairly straight as long as the vertex is rounded.

(b)
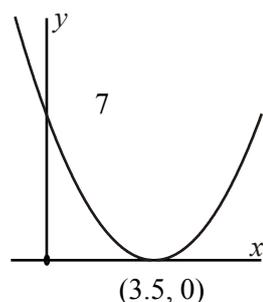
$(3.5, 0)$ B1B1   2

1st B1   for U shaped curve, touching positive *x*-axis and crossing *y*-axis at (0, 7)[condone (7, 0) if marked on positive *y* axis] or 7 marked on *y*-axis

2nd B1   for minimum at (3.5, 0) or 3.5 or $\dfrac{7}{2}$ or marked on *x*-axis.

Do <u>not</u> condone (0, 3.5) here.
Redrawing f(*x*) will score B1B0 in part (b).
Points on sketch override points given in text/table.
If coordinates are given elsewhere (text or table) marks can be awarded if they are compatible with the sketch.

Allow "stopping at" (0, 10) or (0, 7) instead of "cutting"

[5]

7.   (a)
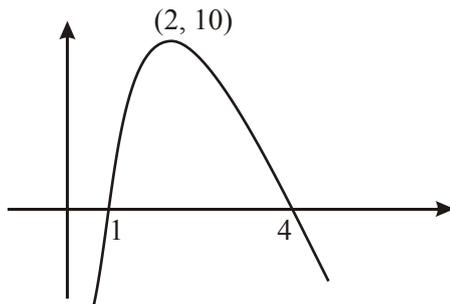
Shape: Max in 1st quadrant and 2 intersections on positive *x*-axis   B1

1 and 4 labelled (in correct place) or clearly stated as coordinates   B1

(2, 10) labelled or clearly stated   B1   3

(b)
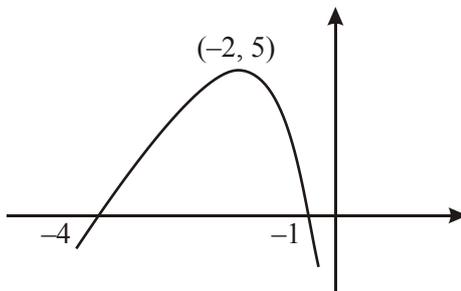
Shape: Max in 2nd quadrant and 2 intersections on negative *x*-axis B1

−1 and −4 labelled (in correct place) or clearly stated as coordinates B1

(−2, 5) labelled or clearly stated   B1   3

Special case:
If the graph is reflected in the *x*-axis (instead of the *y*-axis),
B1 B0 B0 can be scored. This requires shape and coordinates to
be <u>fully correct</u> i.e.

Shape: ____/ Minimum in 4th quadrant and 2 intersections

on positive *x*-axis, 1 and 4 labelled (in correct place) or clearly
stated as coordinates, (2, −5) labelled or clearly stated.

(a) and (b)

1st B: 'Shape' is generous, providing the conditions are satisfied.

2nd and 3rd B marks are dependent upon a sketch having been drawn.

2nd B marks: Allow (0, 1), etc. (coordinates the wrong way round) if the
                       sketch is correct.

Points must be labelled correctly and be in appropriate place
(e.g. (−2, 5) in the first quadrant is B0).

(c)   (*a* =) 2   May be implicit, i.e. f (*x* + 2)   B1   1

[7]

8. (a)
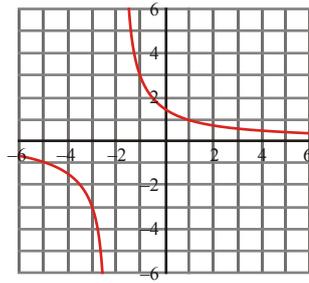
Translation parallel to *x*-axis	M1	
Top branch intersects + ve *y* – axis ⎫		
Lower branch has no intersections ⎬	A1	
No obvious overlap ⎭		
$\left(0, \dfrac{3}{2}\right)$ or $\dfrac{3}{2}$ marked on *y*-axis	B1	3

M1 for a horizontal translation – two branches with one branch cutting *y*-axis only.
If one of the branches cuts both axes (translation up and across) this is M0.

A1 for a horizontal translation to left.
Ignore any figures on axes for this mark.

B1 for correct intersection on positive *y*-axis.
More than 1 intersection is B0.

x = 0 and *y* = 1.5 in a table alone is sufficient unless intersection of their sketch is with +ve *y*-axis.
A point marked on the graph overrides a point given elsewhere.

(b) *x* = –2, *y* = 0 B1, B1 2

S.C. [Allow ft on first B1 for *x* = 2 when translated "the wrong way" but must be compatible with their sketch.]

1st B1 for *x* = –2. NB *x* ≠ –2 is B0.
Can accept *x* = +2 if this is compatible with their sketch.
Usually they will have M1A0 in part (a) (and usually B0 too)

2nd B1 for *y* = 0.

S.C. If *x* = –2 and *y* = 0 and some other asymptotes are also given award B1B0

The asymptote equations should be clearly stated in part (b).
Simply marking *x* = 2 or *y* = 0 on the sketch is sufficient <u>unless</u> they are clearly marked "asymptote *x* = –2" etc.

[5]

9. (a)
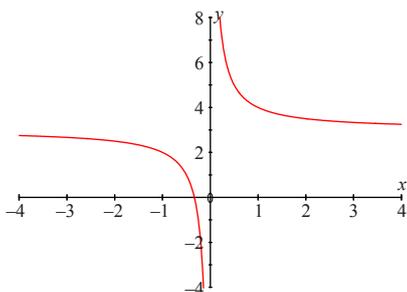
Shape of f(x)	B1
Moved up ↑	M1
Asymptotes: $y = 3$	B1
$x = 0$ (Allow " y –axis")	B1 4

($y \neq 3$ is B0, $x \neq 0$ is B0).

B1: Shape requires both branches and no obvious "overlap" with the asymptotes (see below), but otherwise this mark is awarded generously.
The curve may, e.g., bend away from the asymptote a little at the end.
Sufficient curve must be seen to suggest the asymptotic behaviour, both horizontal and vertical.

M1: Evidence of an upward translation parallel to the y-axis. The shape of the graph can be wrong, but the complete graph (both branches if they have 2 branches) must be translated upwards. This mark can be awarded generously by implication where the graph drawn is an upward translation of another standard curve (but <u>not</u> a straight line).

The B marks for asymptote equations are independent of the graph. Ignore extra asymptote equations, if seen.

(b) $\dfrac{1}{x} + 3 = 0$ No variations accepted.   M1

$x = -\dfrac{1}{3}$ (or -0.33 …)Decimal answer requires at least 2 d.p.   A1   2

Correct answer with no working scores both marks.
The answer may be seen on the sketch in part (a).
Ignore any attempts to find an intersection with the y-axis.

e.g.
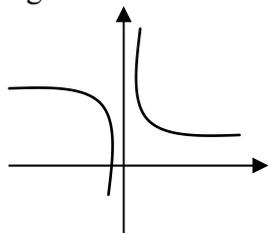
(a)   This scores B0 (<u>clear</u> overlap with horiz. asymp.)
M1 (Upward translation… bod that both branches have
been translated).
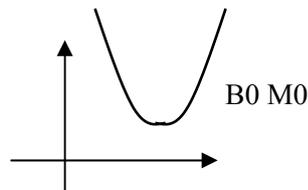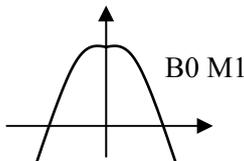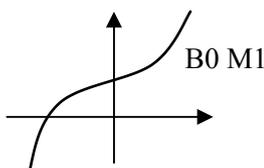
No marks unless the original curve is seen, to show upward translation.

[6]

10.   (a)
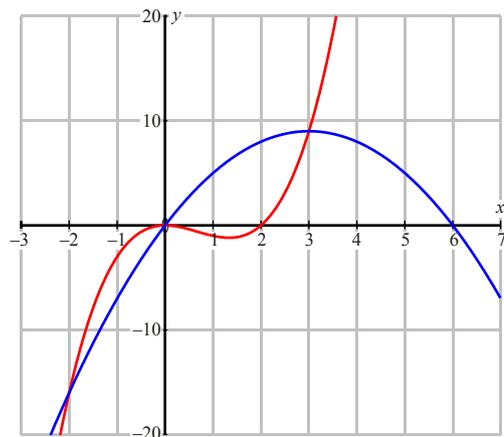
(i)   Shape or or   B1
Max. at (0, 0).   B1
(2, 0), (or 2 shown on *x*-axis).   B1   3

For the third 'shape' shown above, where a section of the
graph coincides with the *x*-axis, the B1 for (2, 0) can still
be awarded if the 2 is shown on the *x*-axis.

For the final B1 in (i), and similarly for (6, 0) in (ii):
There must be a sketch.
If, for example (2, 0) is written <u>separately</u> from the sketch,
the sketch must not clearly contradict this.
If (0, 2) instead of (2, 0) is shown <u>on the sketch</u>, allow the mark.
Ignore extra intersections with the *x*-axis.

(ii) Shape   B1

(It need not go below *x*-axis)

Through origin.   B1

(6, 0), (or 6 shown on *x*-axis).   B1   3

2nd B is dependent on 1st B.

Separate sketches can score all marks.

(b) $x^2(x - 2) = x(6 - x)$ M1

$x^3 - x^2 - 6x = 0$

Expand to form 3-term cubic (or 3-term quadratic if divided by *x*),

with all terms on one side. The "= 0" may be implied.   M1

$x(x - 3)(x + 2) = 0$ $x = ...$Factor *x* (or divide by *x*), and solve quadratic. M1

$x = 3$ and $x = -2$

$x = -2 : y = -16$ Attempt *y* value for a non-zero *x* value by   M1

substituting back into $x^2(x - 2)$ or $x(6 - x)$.

$x = 3 : y = 9$ Both *y* values are needed for A1.   A1

(–2,–16) and (3,9)

(0, 0) This can just be written down. Ignore any 'method' shown.

(But must be seen in part (b)).   B1   7

Note the dependence of the first three M marks.

A common wrong solution is (–2, 0), (3, 0), (0, 0),

which scores M0 A0 B1 as the last 3 marks.

A solution using <u>no</u> algebra (e.g. trial and error), can score up to 3 marks:

M0 M0 M0 A0 M1 A1 B1.

(The final A1 requires both *y* values).

Also, if the cubic is found but not solved algebraically, up to 5 marks:

M1 M1 M0 A0 M1 A1 B1. (The final A1 requires both *y* values).

[13]

11. (a)
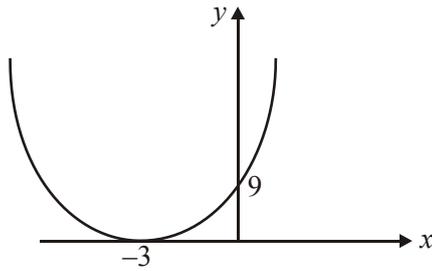
U shape touching x-axis	B1	
$(-3, 0)$	B1	
$(0, 9)$	B1	3

2nd B1   *They can score this even if other intersections with the x-axis are given.*

2nd B1 & 3rd B1   *The -3 and 9 can appear on the sketch as shown*

(b)
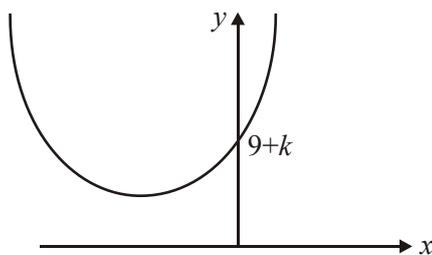
Translated parallel to y-axis up	M1	
$(0, 9 + k)$	B1f.t.	2

    M1   *Follow their curve in (a) up only.*
                 If it is not obvious do not give it. e.g. if it
                 cuts y-axis in (a) but doesn't in (b) then it is M0.
    B 1f.t.   *Follow through their 9*

[5]

12.   (a)
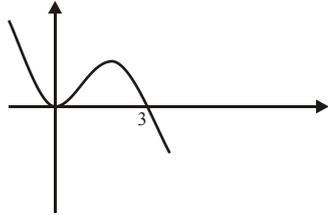
(See below)	M1	
Clearly through origin (or (0, 0) seen)	A1	
3 labelled (or (3, 0) seen)	A1	3

(b)
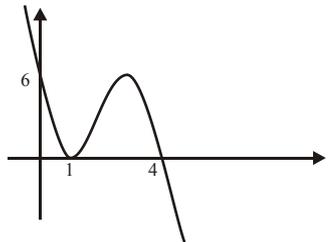
Stretch parallel to *y*-axis	M1	
1 and 4 labelled (or (1, 0) and (4, 0) seen)	A1	
6 labelled (or (0, 6) seen)	A1	3

(c)
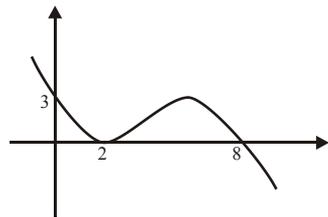
Stretch parallel to *x*-axis	M1	
2 and 8 labelled (or (2, 0) and (8, 0) seen)	A1	
3 labelled (or (0, 3) seen)	A1	3

[9]

(a)   M1:

(b)   M1:

with at least two of: (1, 0) unchanged
                          (4, 0) unchanged
                          (0, 3) changed

M1:
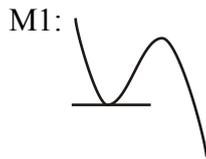
with at least two of: (1, 0) changed
(4, 0) changed
(0, 3) unchanged

<u>Beware</u>: Candidates may sometimes re-label the parts of their solution.

13. (a)
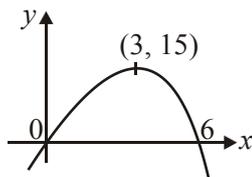
Shape   B1
Points   B1   2

(If both graphs are really straight lines then penalise B0 in part (a) only)

1st B1 for ⌒ shape through (0, 0) and ((k,0) where k > 0)

2nd B1 for max at (3, 15) and 6 labelled or (6, 0) seen
Condone (15,3) if 3 and 15 are correct on axes. Similarly (5,1) in (b)

(b)

M1
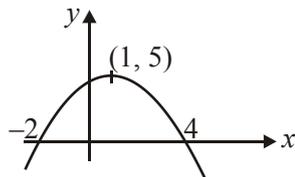
–2 and 4   A1
max A1   3

[5]

Marks for shape: graphs must have curved sides and round top. Don't penalise twice.

M1 for ⌒ shape <u>NOT</u> through (0, 0) but must cut x-axis twice.

1st A1 for –2 and 4 labelled or (–2, 0) and (4, 0) seen

2nd A1 for max at (1, 5). Must be clearly in 1st quadrant

14.   (a)
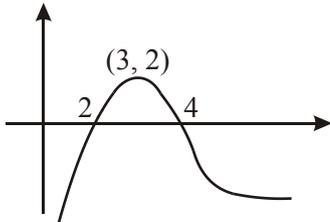
Reflection in *x*-axis, cutting *x*-axis twice.	B1	
2 and 4 labelled (or (2, 0) and (4, 0) seen)	B1}	
Image of *P* (3, 2)	B1	3

   (b)
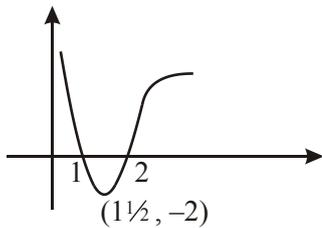
Stretch parallel to *x*-axis	M1	
1 and 2 labelled (or (1, 0) and (2, 0) seen)	A1	
Image of *P* (1½, –2)	A1	3

[6]

15. $x, (x - 2)(x + 2)$ B1, M1 A1   3
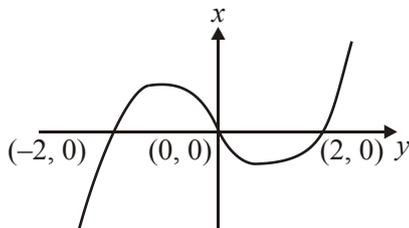
Shape	B1	
Through origin	B1 (dep.)	
–2 and 2	B1	3
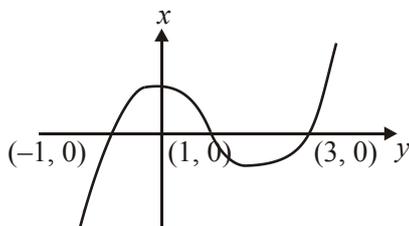
Curve translated +1 parallel to *x*-axis	B1 ft	
–1, 1 and 3 (B1 ft for one value)	B1 ft B1	3

[9]

1.   Most candidates have a good idea of the principles in these transformations (as was evident from their rough working) but sometimes their sketches did not do them justice.

In part (a) many knew that the graph underwent a translation of 3 units to the left but their minimum was in the 4th quadrant even though it was correctly labelled (0, –5).

In (b) many knew that the curve was stretched in the y direction but their curve did not pass through the origin and a mark was lost. The most common error in (a) was a translation of +3 in the x direction and in (b) we saw both $y = f(\frac{1}{2}x)$ and $y = \frac{1}{2}f(x)$.

Part (c) proved to be a little more difficult with candidates unable to visualise the translation and unwilling to draw themselves a diagram. There were many wrong answers of $a = \pm3$ and sometimes $a = -5$.

2.   The majority sketched a quadratic and a cubic curve in part (a) but not always with the correct features. The quadratic was often U shaped and although the intercepts at (0, 0) and (4, 0) were mostly correct, sometimes the curve passes through (–4, 0) and (4, 0). The cubic was sometimes a positive cubic and whilst it often passes through (0, 0) and (7, 0) the turning point was not always at the origin and the intercept was sometimes at (–7, 0).

Part (b) caused few problems with most candidates scoring full marks here.

Most could start part (c) and the quadratic formula was usually used to solve their equation. Although many simplified $\sqrt{48}$ to $4\sqrt{3}$ several candidates failed to divide by 2 correctly and gave their answers as $x = 4 \pm 4\sqrt{3}$. Most realised they needed to find the y-coordinate as well and usually they substituted their value of x into the quadratic equation to find y, though some chose the much less friendly cubic equation instead.

The selection of the correct solution defeated all but the best candidates. Most successful solutions involved checking the y coordinates for both $x = 4 + 2\sqrt{3}$ and $x = 4 - 2\sqrt{3}$ and, if the calculations were correct, selecting the one that gave a positive y coordinate. Only a rare minority realised that the required point would have an x coordinate in the interval (0, 4) and therefore only the $x = 4 - 2\sqrt{3}$ case need be considered.

3.   There were many good solutions to all three parts of this question. Although many candidates were able to give the coordinates of the transformed maximum points correctly, some did not understand the effect of the transformation on the asymptote. This was particularly true in part (b), where it was common to think that the asymptote $y = 1$ was unchanged in the transformation $y = 4f(x)$. Almost all candidates had some success in producing sketches of the correct general shape in each part, but it was often apparent that the concept of an asymptote was not fully understood.

4.   It seemed clear that many candidates did not appreciate the links between the 3 parts of this question and there was much unnecessary work carried out: differentiating to find turning points and tables of values to help draw, not sketch, the curves.

Part (a) was usually answered well but not all the successful candidates started by taking out the factor of *x*, rather they tried to use the factor theorem to establish a first factor. Whilst techniques from C2 (or higher units) **may** be used in C1 they are not required and the "best" approach will not use them.

A correct factorisation in (a) should have made the sketch in (b) straightforward. Most drew a cubic curve (but some had a negative cubic not a positive one) and usually their curve either touched or passed through the origin. The most common non-cubic curve was a parabola passing through (0, 0) and (3,0). Part (c) looked complicated but those who spotted that they were sketching f(*x* – 2) had few problems in securing both marks. Many candidates though embarked upon half a page or more of algebraic manipulation to no avail - this part was only worth 2 marks and a little thought may have helped them realise that such an approach was unlikely to be the correct one.

5.   There were many good solutions to both parts of this question. In part (a) most candidates translated the curve parallel to the *x*-axis, although occasionally the translation was of + 3 rather than – 3 units, taking the curve "to the right". A common mistake in part (b) was to sketch *y* = –f (*x*) instead of *y* = f (–*x*), reflecting in the *x*-axis instead of the y-axis.
Just a few candidates failed to show the coordinates of the turning points or intersections with the *x*-axis, or carelessly omitted a minus sign from a coordinate.

6.   In part (a) most realized that a translation was required but some moved the curve horizontally or downwards. Some forgot to write the coordinates of the new turning point on their curve and a few placed the point (7, 3) on their horizontal axis. In part (b) the stretch was usually identified but curves often crossed at (0, 14) or touched at (14, 0). The shape of the curve was usually "preserved" quite well but sometimes the vertex became too pointed or one end of the curve seemed to bend away towards an extra turning point. A minority of candidates still make errors by writing coordinates the wrong way round.

7.   In part (a), most candidates knew that a stretch was required. It was common to see full marks scored, although the final mark was sometimes lost because the maximum was not labelled. A common wrong answer for the maximum was (4, 10) instead of (2, 10). Other mistakes included stretches in the *x* direction instead of the *y* direction and stretches with scale factor $\frac{1}{2}$ instead of 2.

The most common mistake in part (b) was to reflect in the *x*-axis instead of the *y*-axis (scoring just 1 mark out of 3). It was not unusual in this part to see the required points carelessly mislabelled with minus signs omitted. Many candidates did not answer part (c), but for those that did there were several common wrong answers, particularly 5, – 2 or 3.

8. There were very mixed responses to this question; some answered it very well with neat sketches and clearly identified asymptotes but others appeared to have little idea. In part (a) most candidates knew that a translation was required and the majority knew it was horizontal and to the left. A few moved their graph up and a number of candidates ended up with graphs cutting both axes. Many identified the intersection with the *y*-axis as (0, 1.5) but some ignored this part of the question. It was clear in part (b) that a number of candidates still do not know the meaning of the term asymptote but many did give *x* = – 2 and slightly fewer gave *y* = 0.
    However a number of candidates gave answers of *y* = – 2 and *x* = 0, often despite having a correct dotted line on their sketch, and some gave other non-linear equations such as $y = \dfrac{3}{x}$.
    Some candidates gave no answer to part (b) despite having the lines *x* = – 2 and *y* = 0 clearly identified on their diagram.

9. Disappointingly, a significant number of candidates were unfamiliar with the rectangular hyperbola. Parabolas, cubic curves and straight lines were sometimes sketched. Attempts to sketch by plotting points probably wasted time and often led to unconvincingly shaped graphs. The required translation parallel to the *y*-axis was often performed correctly, although horizontal translations and other variations were sometimes seen. Many candidates were unaware of the concept of an asymptote, and even those who demonstrated understanding were often unable to write down equations of the two asymptotes. The asymptote equation *x* = 0 was often omitted. In part (b), answers for the point of intersection with the *x*-axis sometimes contradicted what was seen in candidates' sketches, but this part was marked independently and many were able to solve $\dfrac{1}{x} + 3 = 0$ correctly. Others, however, were defeated by the algebra here or tried to evaluate $\dfrac{1}{0} + 3$ to find a *y* value.

10. Weaker candidates sometimes made very little progress with this question. In general, however, curve sketching tended to be disappointing in part (a) but sound algebra was often seen in part (b). Most candidates recognised the cubic and drew a curve of the correct shape, but many placed the repeated root at (2, 0) rather than (0, 0). Many cubic curves also passed through the point (–0 ,2). The parabola commonly appeared upside down or on the negative *x*-axis. A significant number of candidates constructed a table of values and plotted points, suggesting a lack of knowledge of the respective families of curves. Plotted points from a table of values were often insufficient to establish the complete shape required. Occasionally these tables were used to find one or more intersection points for part (b).

In part (b), most candidates knew how to form the required equation, but there were occasionally sign errors in the algebraic manipulation. Instead of taking out the common factor x many divided through by x and failed to include (0,0) as one of the intersection points. Apart from slips, the resulting quadratic was usually well solved and, having found two x coordinates, most candidates continued and calculated the corresponding y coordinates. A common calculation mistake at this final stage gave $(-,2 - 8)$ instead of $(-,2 - 16)$, and a few candidates thought that all the y coordinates were zero. In many cases, answers to part (b) contradicted what was seen in candidates' sketches in part (a).

11.   This question was answered well with most candidates having a good idea of the shape and identifying the need for a vertical translation. In part (a) most drew a parabola but some common errors were $y = (x - 3)^2$, $y = x^2 + 9$ and $y = (x - 3)(x + 3)$ Some candidates thought that the curve should be asymptotic to the y-axis. Attempts at part (b) were sometimes less successful. Some could not deal with a general number k, choosing to give it a value, and vertical stretches rather than a translation were quite common. In some cases it was not clear what transformation had been applied.

12.   Many candidates were well prepared for this question, scoring high marks. Answers to part (a) were usually correct, with few translations going 'the wrong way'. In parts (b) and (c), method marks were given for attempted stretches parallel to the y-axis and x-axis respectively but it was common here to see stretches in the wrong direction and two-way stretches. The most popular mistake was to stretch, for $y = f\left(\dfrac{1}{2}x\right)$, by a factor of $\dfrac{1}{2}$ instead of 2.

13.   In part (a) over 90% of the candidates sketched a quadratic curve passing through the origin but the stretch was sometimes in the x-direction. Part (b) was generally answered more successfully than part (a) and the curve was usually moved correctly although a minority translated it to the right.

Coordinates were not always written carefully and (2, 0) or (0, -2) were often seen instead of (-2, 0) in part (b).

14.   Although there were many correct solutions to part (a), a substantial number of candidates thought that $y = -f(x)$ could be represented by a reflection in the y-axis rather than the x-axis. In part (b), a method mark was given for an attempted "stretch" parallel to the x-axis, but other types of transformation were frequently seen, including stretches in the wrong direction, two-way stretches and a variety of translations. Stretching by a factor of 2 instead of $\dfrac{1}{2}$ was a common mistake.

15. No Report available for this question.